Sequences - Convergence (limit)
- Increasing, Decreasing (monotonic)
- Bounded

Series - Add terms of a sequence . . . .

$$\overset{1}{.3} + \overset{2}{.03} + \overset{3}{.003} + \overset{4}{.0003} + .00003 + \cdots$$

$$\sum_{K=1}^{\infty} 3 \cdot 10^{-K} = \sum_{K=1}^{\infty} \frac{3}{10^K} = \frac{1}{3}$$

$.3 \times \frac{1}{K}$ $\quad 3 \times 10^{-n}$ $\quad 3\left(\frac{1}{10}\right)$ $K=1$

$(.3)^{-K}$ $\quad 3^{-K}$ $\quad .3 \times 10^{-n}$ $\quad 3\left(\frac{1}{10^2}\right)$ $K=2$

$3\left(\frac{1}{10^3}\right)$ $K=3$

$$10\,(S = .3 + .03 + .003 + .0003 + \cdots)$$

$$10S = 3 + (.3 + .03 + .003 + \cdots)$$

$$10S = 3 + S$$
$$-S \qquad -S$$
$$9S = 3$$
$$S = \frac{3}{9} = \frac{1}{3}$$

$$S = \sum_{k=1}^{\infty} \left(\frac{3}{2}\right)^k \qquad S = -3$$

$$S = \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \cdots$$

$$S = \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \cdots$$

$$\frac{2}{3}S = 1 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^2 + \cdots$$

$$\frac{2}{3}S = 1 + S$$

$$-\frac{1}{3}S = 1$$

$$\underline{S = -3}$$

$$\frac{3}{2}S = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \cdots$$

$$-\left(S = \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \cdots\right)$$

$$\frac{1}{2}S = -\frac{3}{2} + 0 + 0 + 0 + 0 + \cdots$$

$$\frac{1}{2}S = -\frac{3}{2}$$

$$S = -3$$

$$\frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \cdots$$

① $\frac{3}{2} \approx 1.5 = \sum_{k=1}^{1} \left(\frac{3}{2}\right)^k$ .3

② $\frac{15}{4} \approx 3.75 = \sum_{k=1}^{2}$ .33

.333

③ $\approx 7.125 = \sum_{k=1}^{3}$ .3333

$\vdots$

PARTIAL SUMS

Sequences - Convergence (limit)

- Increasing, Decreasing (Monotonic)
- Bounded

ARITHMETIC Σ GEOMETRIC Σ

Series: add up the terms of sequence

K: 1, 2, 3, 4 . . .

$.3 + .03 + .003 + .0003 + .00003 + \cdots$

$$\sum_{K=1}^{\infty} (3 \cdot 10^{-K}) = \sum_{K=1}^{\infty} \frac{3}{10^K} \cdots$$

3×10^{-K}

$10(S = .3 + .03 + .003 + .0003 + \cdots)$

$10S = 3 + (.3 + .03 + .003 + \cdots)$

$10S = 3 + S$

$9S = 3$

$S = \frac{1}{3}$

$$\sum_{K=1}^{\infty} \left(\frac{3}{2}\right)^K = S$$

$$S = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \ldots\ldots$$

$$\frac{2}{3}\left(S = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \ldots\right)$$

$$\frac{2}{3}S = 1 + (S)$$

$$-S$$

$$-\tfrac{1}{3}S = 1$$

$$S = -3$$

$$S = \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \ldots$$

$$\frac{3}{2}\left(S = \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \ldots\right)$$

$$\frac{3}{2}S = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \ldots$$

$$S - \frac{3}{2}S = \frac{3}{2} + 0 + 0$$

$$-\frac{1}{2}S = \frac{3}{2} \Rightarrow S = -3$$

$$S = \sum_{k=1}^{\infty} \left(\frac{3}{2}\right)^k$$

$$= \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \cdots$$

$S_1 = 1$ $\quad \frac{3}{2} = 1.5$ $\quad 1 \quad .3$

$S_2 = 2$ $\quad \frac{15}{4} = 3.75$ $\quad 2 \quad .33$

$S_3 = 3$ $\quad \frac{57}{8} = 7.125$ $\quad 3 \quad .333$

$\vdots \quad \vdots \quad \vdots$

PARTIAL SUMS

Series

$\frac{1}{10}\left(\frac{1}{1-\frac{1}{10}}\right) = \frac{1}{10}\left(\frac{1}{\frac{9}{10}}\right) = \frac{1}{10}\left(\frac{10}{9}\right) = \frac{1}{9}$

$\sum_{K=1}^{\infty}\left(\frac{1}{10}\right)^K = .11111 = \frac{1}{9}$ converges | $\sum_{K=1}^{\infty}\left(\frac{3}{2}\right)^K$ diverges

A series $\sum_{K=1}^{\infty} a_K$ converges if

The sequence of PARTIAL sums converges

S_1 = first partial sum $= \sum_{K=1}^{1} a_K$

S_2 = second " " $= \sum_{K=1}^{2} a_K$

...

S_n = n^{th} " " $= \sum_{K=1}^{n} a_K$

If $\sum_{K=1}^{\infty} a_K$ converges then $\lim_{K\to\infty} a_K = 0$

Logic

If a student is a freshman, then that student is short. ✓

is: If a student is short, then that student is a freshman. CONVERSE

is: if a student is NOT short, then that student is NOT a freshman. CONTRAPOSITIVE

★ If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges.

6) $\sum_{K=3}^{\infty} (-1)^K \left(\frac{3}{2^K}\right) = \sum_{K=3}^{\infty} \frac{(-1)^K}{2^K}(3)$

$= \sum_{K=3}^{\infty} \left(\frac{-1}{2}\right)^K 3 =$

$3 \sum_{K=3}^{\infty} \left(\frac{-1}{2}\right)^K$

$2\left(\frac{3}{5}\right) = \frac{2}{5}(3)$

$= 3\left(-\frac{1}{2}\right)^3 \sum_{K=0}^{\infty} \left(\frac{-1}{2}\right)^K = 3\left(-\frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} \cdots\right)$

$= 3\left(\frac{-1}{2}\right)^3 \left(\frac{1}{1-\left(\frac{-1}{2}\right)}\right)$

$\sum_{g=0}^{\infty} ar^g = a + ar + ar^2 \ldots$

$a\left(\frac{1}{1-r}\right)$

$= 3\left(\frac{-1}{8}\right)\left(\frac{1}{\frac{3}{2}}\right) = 3\left(\frac{-1}{8}\right)\left(\frac{2}{3}\right)$

$= \frac{-2}{8} = \frac{-1}{4}$

$= 3\left(-\frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} \cdots\right)$

$-\frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} + \cdots$

$= -\frac{1}{8}\left(1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots\right)$

7) $\sum_{k=1}^{\infty} \frac{4}{k(k+2)} = 4 \sum_{k=1}^{\infty} \frac{1}{k(k+2)}$

$$\frac{1}{k(k+2)} = \frac{A}{k} + \frac{B}{k+2}$$

$$= \frac{A(k+2)+B(k)}{k(k+2)}$$

$A(k+2)+B(k) = 1 \Rightarrow Ak+2A+Bk=1$

$k(A+B)+2A = 1+0k$

$\Downarrow$

$A+B=0$

$2A=1$

when $k=0$ . . .

$2A=1 \Rightarrow A=\frac{1}{2}$

when $k=-2$

$-2B=1 \Rightarrow B=-\frac{1}{2}$

$$4\sum_{k=1}^{\infty} \frac{1}{k(k+2)} = 4\sum_{k=1}^{\infty}\left[\frac{1}{2}\left(\frac{1}{k}\right) - \frac{1}{2}\left(\frac{1}{k+2}\right)\right]$$

$$= 4\Big(\frac{1}{2}\left(\frac{1}{1}\right) - \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3}\right) - \frac{1}{2}\left(\frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{6}\right) \ldots$$

$$= 4\left(\frac{1}{2} + \frac{1}{4} + 0\left(\frac{1}{3}\right) + 0\left(\frac{1}{4}\right) + 0 + 0 + 0 + 0 + \ldots\right.$$

$$= 4\left(\frac{3}{4}\right) = 3$$

Series

$$\sum_{K=1}^{\infty} \frac{3}{10^K}$$ converges to $\frac{1}{3}$

$$\sum_{K=1}^{\infty} \left(\frac{3}{2}\right)^K$$ diverges

~~In general~~ If the series $\sum_{K=1}^{\infty} a_K$ CONVERGES
Then ~~means that~~ $\lim_{K\to\infty} a_K = 0$

Logic

if a student is a freshman, then the student is short.

is true? If a student is short, then the student is a freshman. CONVERSE

is true? if a student is NOT short, Then the student is NOT a freshman. CONTRA-POSITIVE

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges.

Reminder A series $\sum_{K=1}^{\infty} a_K$ converges if the sequence of PARTIAL SUMS converges.

8.2 2)

$$\sum_{K=0}^{\infty} \frac{1}{3}(5^K)$$

from divergence test, diverges.

(4) $$\sum_{K=0}^{\infty} 4\left(\frac{1}{2}\right)^K = 4\sum_{K=0}^{\infty}\left(\frac{1}{2}\right)^K$$

$$= 4\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$$

$$S = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$$

$$-\left(\frac{1}{2}S = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$$

$$\frac{1}{2}S = 1$$

$$\therefore S = 2$$

$$\sum_{K=0}^{\infty} 4\left(\frac{1}{2}\right)^K$$

$$= 4\sum_{K=0}^{\infty}\left(\frac{1}{2}\right)^K$$

$$= 4\left(\frac{1}{1-\frac{1}{2}}\right) = 4\cdot 2 = 8$$

$$\sum_{K=0}^{\infty} ar^K = a\left(\frac{1}{1-r}\right)$$

assuming $|r|<1$

if $|r|\geq 1$ then diverges.

$$\sum_{K=0}^{\infty} ar^K = a+ar+ar^2+\cdots$$

$$S = a+ar+ar^2+\cdots$$

$$rS = ar+a^2+\cdots$$

$$S-rS = a$$

$$(1-r)S = a$$

7* $\sum_{K=1}^{\infty} \frac{1}{K(K+2)} = \frac{3}{4}$

$$\frac{1}{K(K+2)} = \frac{\cancel{A}\ \frac{1}{2}}{K} + \frac{B}{K+2}$$

$$= \frac{A(K+2)+B(K)}{K(K+2)}$$

$$A(K+2)+B(K)=1$$

$AK+2A+BK=1$

$(A+B)K+2A=0K+1$

$A+B=0$

$2A=1$

when $K=0$

$2A=1$

$\underline{A=\frac{1}{2}}$

when $K=-2$

$-2B=1$

$B=-\frac{1}{2}$

$$\frac{1}{K(K+2)} = \frac{1}{2}\left(\frac{1}{K}\right)+\left(-\frac{1}{2}\right)\left(\frac{1}{K+2}\right)$$

$$\sum_{K=1}^{\infty} \frac{1}{K(K+2)} = \sum_{K=1}^{\infty} \frac{1}{2}\left(\frac{1}{K}\right)-\frac{1}{2}\left(\frac{1}{K+2}\right)$$

$$= \frac{1}{2}(1) - \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3}\right) - \frac{1}{2}\left(\frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{4}\right) \ldots$$

$=0$ $=0$ $=0$ $=0$

$$= \frac{1}{2}+\frac{1}{4}$$